INFOMGP - GAME PHYSICS

EXERCISES LECTURE 5

EXERCISE 5.1

Suppose you have an object at $t = 0$ second sitting still at the origin. Its mass is 1 kg and the net force applied on it is $F(t) = \begin{pmatrix} 0 \\ t+1 \end{pmatrix}$. Find the position of the object after 1, 2 and 3 seconds using Euler's method.

Iteration 1:

$$a(0) = \frac{F(0)}{m} = F(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$v(1) = v(0) + a(0) \times 1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \times 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$p(1) = p(0) + v(0) \times 1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \times 1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Iteration 2:

$$a(1) = \frac{F(1)}{m} = F(1) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$
$$v(2) = v(1) + a(1) \times 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \times 1 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$
$$p(2) = p(1) + v(1) \times 1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \times 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Iteration 3:

$$a(2) = \frac{F(2)}{m} = F(2) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$
$$v(3) = v(2) + a(2) \times 1 = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} \times 1 = \begin{pmatrix} 0 \\ 6 \end{pmatrix}$$
$$p(3) = p(2) + v(2) \times 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} \times 1 = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

EXERCISE 5.2

Assuming an object is decelerated by a drag force of $a(t, v) = -v$ and at $t = 0$ second the velocity of the object is 20 m/s. What will be the velocity of the object after 0.5 second?
Calculate $v(t + \Delta t)$ with Euler's method, the midpoint method, the improved Euler's method and RK4 method.
Then compare the results with the ideal solution (*hint:* $\int dv = \int -v(t)dt \iff v(t) = v(0)e^{-t}$).

Euler's method

$$v(t + \Delta t) = v(t) + \Delta t\ a(t, v) = 20 + 0.5 \times (-20) = 10 \text{ m/s}$$

Midpoint method

$$v\left(t + \frac{\Delta t}{2}\right) = v(t) + \frac{\Delta t}{2} a(t, v) = 20 + 0.25 \times (-20) = 15$$

$$v(t + \Delta t) = v(t) + \Delta t\ a\left(t + \tfrac{\Delta t}{2}, v\left(t + \tfrac{\Delta t}{2}\right)\right) = 20 + 0.5 \times (-15) = 12.5 \text{ m/s}$$

Improved Euler's method

$$v_1 = v(t) + \Delta t\ a(t, v) = 10$$

$$v_2 = v(t) + \Delta t\ a(t + \Delta t, v_1) = 20 + 0.5 \times (-10) = 15$$

$$v(t + \Delta t) = \tfrac{v_1 + v_2}{2} = 12.5 \text{ m/s}$$

RK4

$$v_1 = \Delta t \times a\big(t, v(t)\big) = 0.5 \times (-20) = -10$$

$$v_2 = \Delta t \times a\left(t + \frac{\Delta t}{2}, v(t) + \frac{1}{2} v_1\right) = 0.5 \times \left(-\left(20 - \frac{1}{2} 10\right)\right) = -10 + \frac{10}{4} = -7.5$$

$$v_3 = \Delta t \times a\left(t + \frac{\Delta t}{2}, v(t) + \frac{1}{2} v_2\right) = 0.5 \times \left(-\left(20 - \frac{1}{2} 7.5\right)\right) = -10 + \frac{7.5}{4} = -8.125$$

$$v_4 = \Delta t \times a(t + \Delta t, v(t) + v_3) = 0.5 \times (-(20 - 8.125)) = -10 + \frac{8.125}{2} = -5.9375$$

$$v(t + \Delta t) = v(t) + \tfrac{v_1 + 2v_2 + 2v_3 + v_4}{6} \approx 20 - 7.8645 \approx 12.135 \text{ m/s}$$

Ideal

$$a = \frac{dv}{dt} = -v(t) \Leftrightarrow \int dv = \int -v(t) dt \Leftrightarrow v(t) = v(0) e^{-t}$$

$$v(t + \Delta t) = v(0 + 0.5) = 20 e^{-0.5} \approx 12.131 \text{ m/s}$$